Calculus AB

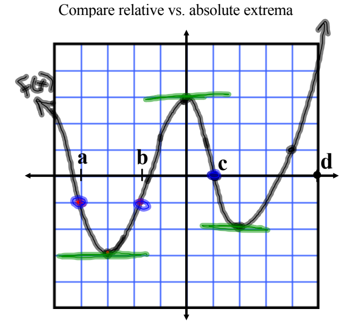
3-1

Extrema on an Interval

Extrema- Maximum and Minimum values of a function on a given interval.

	min	max
[a,b]	-3	-1
[b,c]	-1	3
[c,d]	-2	4
[a,c]	-3	3
[b,d]	-2	4
[a,d]	-3	4
(a,b)	-3	none
(b,c)	none	3
(c,d)	-2	none
(a,d)	-3	none
all Reals	-3	∞ ← absolute

Compare relative vs. absolute extrema



Extreme Value Theorem- If f is continuous on [a,b], then f has both a minimum and a maximum on the interval.

Critical Point- A point on a graph where the derivative is equal to zero or does not exist.

Theorem- Relative Extrema only occur at critical points.

Find any critical numbers of the function. (pg 169)

12) $g(x) = x^4 - 4x^2$

$g'(x) = 4x^3 - 8x$

$0 = 4x(x^2 - 2)$ or does not exist

$4x = 0 \qquad x^2 - 2 = 0$

$x = 0 \qquad \sqrt{x^2} = \sqrt{2}$

$|x| = \sqrt{2}$

$x = \pm\sqrt{2}$

$\boxed{0, \pm\sqrt{2}}$

none here

The derivative is continuous with no asymptotes, radicals, fractions, etc. It is defined on $\mathbb{R}$

Locate the absolute extrema of the function on the closed interval.

20) $h(x) = -x^2 + 3x - 5$, on [-2, 1]

$h'(x) = -2x + 3$

$0 = -2x + 3$

$x = \frac{3}{2}$ Not in

Since 3/2 is not in the given interval of [-2,1], we cannot consider it for extrema.

$\boxed{\text{check critical pts, end pts}}$

$h(-2) = -15$ min

$h(1) = -3$ max

Locate the absolute extrema of the function on the closed interval.

34) $f(x) = \tan\left(\frac{\pi x}{8}\right)$, on [0, 2]

$f'(x) = \frac{\pi}{8}\sec^2\left(\frac{\pi}{8}x\right)$

$\sqrt{0} = \sqrt{\cancel{\frac{\pi}{8}}\sec^2\left(\frac{\pi}{8}x\right)}$

$\pm 0 = \sec\left(\frac{\pi}{8}x\right)$

Never

undefined at $4 + 8n$, $n \in \mathbb{Z}$

$-12, -4, 4, 12$ etc

Why?

period $\frac{2\pi}{b}$

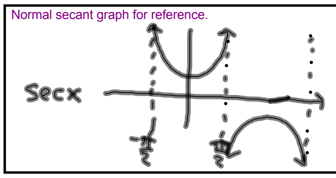
$\frac{2\pi}{\frac{\pi}{8}} = 2\pi \cdot \frac{8}{\pi} = 16$

Since the period is 16, and secant has two sets of asymptotes per period, the asymptotes are 8 units apart.

$f(0) = 0$ min

$f(2) = 1$ max

Normal secant graph for reference.



61) The formula for the power output P of a battery is $P = VI - RI^2$ where V is the electromotive force in volts, R is the resistance, and I is the current. Find the current (measured in amperes) that corresponds to a maximum value of P in a battery for which $V = 12$ volts and $R = 0.5$ ohms. Assume that a 15-amp fuse bounds the output in the interval for I of [0,15]. Could the power output be increased by replacing the 15-amp fuse with a 20 amp fuse?

$P = VI - RI^2$

$P = 12I - 0.5I^2$

$\frac{dP}{dI} = 12 - 1 \cdot I$

$0 = 12 - I$

$I = 12$

ep (0,0)

cp (12,72) max

ep (15,67.5)

for 20 amp fuse → 20,40

Since 72 is the max power at 12 volts, replacing the 15 amp fuse with the 20 amp fuse would not increase power.

Assignment:

Pg. 169

11-35 odd,

39, 41, 43,

54, 57-60